Using the Van Hiele phases of Instruction to Design and Implement a Circle Geometry Teaching Programme in a Secondary School in Oshikoto Region: A Namibian Case Study

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Abstract

Despite my positive views of geometry as one of the most interesting topics in the mathematics curriculum, others find it rather complicated and sometimes unbearable to successfully complete a geometry course. It is a common phenomenon that secondary school students globally struggle with simple geometric problems and their thinking and reasoning are generally below average. Although difficulties in geometric thinking and reasoning are prevalent in global educational research, problems in geometric conceptualisation still prevail. It is therefore the aim of this paper to report on the case study research that was conducted to examine, analyse and report on the findings of the experiences of selected mathematics teachers when they used the Van Hiele phases of instruction in designing and implementing a Grade 11 circle geometry teaching programme. The sample consisted of three selected Grade 11 mathematics teachers from the school where the researcher taught. The findings of this research revealed that teachers used and implemented all the five Van Hiele phases of instruction in their lessons whilst navigating quite freely from one phase of instruction to the next, but also returned to the earlier phases for clarification and reinforcement in their teaching. Teachers also saw the phases of instruction as a good pedagogical tool or template for planning and presenting lessons. The majority of the learners followed the instructions and seemed to obtain the answers faster than expected.

Keywords

Geometry, circle geometry, teaching programme, Van Hiele phases, teachers' experiences

Introduction

The mathematics teachers' approach to geometric instruction determines to a large extent the mathematical thinking strategies and dispositions that our learners attain and develop. In my experience as a mathematics teacher, I have observed how many high school students struggle to recognise, make accurate constructions, accurately describe properties of plane geometry figures, and construct appropriate proofs in geometry. I have noted with frustration how students often reached a dead-end when it comes to solving geometrical problems. Some even believe that geometry is the most difficult aspect of mathematics which in turn creates a feeling of resentment towards geometry as a mathematical activity.

In my experience, mathematics teachers in Namibia highlight numerous factors that they believe contribute to their students' poor performance and failure in geometry. These constraining factors include an inappropriate curriculum, weak textbooks, lack of teaching/learning aids and unmotivated students. Van Hiele's (1986) solution to overcoming these problems is for teachers to take responsibility for their own teaching and to make appropriate choices. For example, if the curriculum is not suitable for your learners, design your own and if the textbook is inappropriate for teaching and learning in your classroom, restructure it to suit your environment.

The difficulties that learners experience with geometric conceptualisation arise from various factors, but their inability to reason at a higher level of geometric thinking does not lie solely within their own learning ability or motivation. The teacher's instructions and choice of exercises also play an integral role in the pupils' learning. The renowned Dutch mathematics educator and researcher, Van Hiele (1986), suggests that there are difficult moments that face every high school teacher in teaching geometry. He too encountered many moments where pupils failed to understand his teaching. According to Van Hiele (1986, p. 45), "learning mathematics meant learning to think, and to be able to think precisely you should have attained the highest possible level". How then can mathematics teachers ensure that their pupils attain the highest possible level if the teachers themselves are not certain about what levels the pupils have achieved? While investigating this teaching challenge, I was particularly inspired by the Van Hiele (1958, 1959, 1986, 1999) theory that sought to provide answers to the teaching and learning of geometry in high schools. "The Van Hiele theory is a learning model that describes the geometric thinking students go through as they move from a holistic perception of geometric shapes to a refined understanding of geometric proof" (Genz, 2006, p. 4).

Literature Review

Problems with geometry teaching and Learning

"There is no king road to geometry"

That was the answer that Euclid gave to the king of Egypt when he asked him to explain his elements in an easier way. (Dimakos, Nikoloudakis, Ferentinos & Choustoulakis, 2007, p. 90). This illustrates that the difficulties in understanding geometry are not unusual, they have existed since ancient times. Nikoloudakis (2009) observes that similar research on the understanding of geometric concepts by learners has shown that learners in general find defining and recognising geometric shapes and the use of deductive thinking in geometry problematic (p. 17). Burger and Shaughnessy (1986) echo this sentiment in their study where a number of secondary school learners interviewed in their clinical study had incomplete notions of basic shapes and their properties. "This observation might explain some of the frustrations students and teachers have with secondary school geometry courses. Students are not sufficiently grounded in basic geometry concepts and relation to 'reinvent' Euclidean geometry. Memorization may be their only recourse" (p. 46)

Many other researchers have also reported on learners' geometry related difficulties in their various fields of study. Weber (2003, as cited in Nikoloudakis, 2009) found that learners found it very difficult to successfully write simple geometry proofs. Senk (1989), on the other hand states that many secondary school learners in the United States were not prepared for geometry classes. Fuys Geddes and Tischler., (1988) found that too much emphasis was placed on formal symbolism and identification in the elementary school geometry curriculum, while relational understanding was underestimated. Research done in Southern Africa revealed similar problems when it came to assessing why learners struggled with formal geometry.

The De Villiers and Njisane (1987, as cited in de Villiers, 1996, p. 12) study revealed that about 45% of learners in Grade 12 (Std 10) in KwaZulu Natal had only mastered Level 2 or lower, whereas the examination assumed mastery at Level 3 and beyond. De Villiers (1996) further attributes the failure of geometry in many secondary schools to the role of language that creates communication gaps between the teacher and the learners. Atebe and Schafer (2011) in their study showed that participating secondary school learners "had a limited and arguably inadequate knowledge of basic geometric terminology …" (p. 63). This can also be attributed to the traditional curriculum which is typically presented at a higher level than those of the learners (Van Hiele, 1986).

Like in South Africa (De Villiers, 2010), Namibia has a geometry curriculum in secondary schools that is heavily loaded with formal geometry. I often feel frustrated at the lack of geometry knowledge and experience many of the lower secondary learners bring from the primary school. In my view, learners in the primary schools do not spend enough time dealing with geometric ideas in a conceptual manner – their geometric understanding is often shallow and lacks conceptual understanding.

Research revealed that difficulties with geometric conceptualisation are often a result of various factors. Learners' apparent inability to reason at a higher level of geometric thinking does not only lie within their own learning patterns or motivation. The teacher's instructions and choice of exercises also play an integral and important role in the learners' learning. Burger and Shaughnessy (1986) explain that high school geometry as it is taught in most high schools is taught at a deductive level, while most learners are only capable of reasoning informally about geometric concepts upon entrance into geometry. De Villiers (2010) echoes this sentiment and states that within the South African context the main reason for the failure of the traditional geometry curriculum is because its expectations are set at a higher level than that of the learners' ability. For example, the curriculum might require the learners to reason at Van Hiele's level three of geometric understanding while the learners are only able to reason up to the second Van Hiele level.

In my experience as a mathematics teacher, I have observed that many of my high school learners struggle to recognise shapes, make accurate constructions, accurately describe properties of plane geometry figures, and construct appropriate proofs in geometry. This leads to a negative view of geometry as a mathematical activity that might affect all learners, even those who are mathematically oriented. I, for example, conversed with a Computer Studies specialist (a Peace Corps volunteer) who helped out with one Grade 8 mathematics class during his time at my school. I asked him how he felt about geometry in general, and this was his response: "I hate geometry! I am an Algebra guru." Do not get me wrong, this teacher has a sound knowledge of formal geometry but only 'hates' geometry "because it is too complicated or rather people like to complicate it instead of just calling a spade a spade" (Personal communication, Q. Lee, July 12, 2012). The question I ask is whether geometry is really that complicated or could there be other factors that contribute towards 'geometric' confusion among individuals.

Despite many curriculum reforms over the decades the geometry curriculum remains inaccessible to many learners. The renowned Dutch mathematics educator and researcher, Van Hiele (1986), discovered that there were difficult moments that faced every high school teacher in teaching geometry. He too encountered many such moments when learners failed to understand his teaching. Thus, this study was designed specifically to look for possible ways of teaching geometry so that it facilitates the attainment of higher Van Hiele levels. The teaching intervention that is the central part of this study is intended to inspire and encourage teachers to design their own curriculum in such a way that it enables learners to develop meaningful geometric thinking.

Therefore, like teaching any other topic in mathematics, teachers should develop geometric concepts in such a way that learners are able to work from the less abstract to the more complex materials. This involves a carefully planned sequence of teaching events that develop geometric conceptualisation. The Van Hiele theory provides a model to do just that. The rationale and aspiration that underpin the literature reviewed for this study were not only anchored in my personal experience as a mathematics teacher, but have been inspired by the Van Hiele theory (Atebe & Schafer, 2008, 2010, 2011; De Villiers, 1987, 2010; Van Hiele, 1958, 1959, 1986, 1999;) that sought to provide answers to the teaching and learning problems of geometry in secondary schools in Namibia and beyond.

If learning geometry means learning to think and being able to attain the 'highest possible level' of conceptualisation (Van Hiele, 1986), then all mathematics teachers should be well versed in the nature of good geometry teaching.

Conceptual Framework - The Van Hiele Theory

The Van Hiele theory is a "learning model that describes the geometric thinking students should go through as they move from a holistic perception of geometric shapes to a refined understanding of geometric proof" (Genz, 2006, p. 4). This theoretical model was developed by the renowned educators, Pierre Marie van Hiele and his wife Dina Van Hiele-Geldof about six decades ago. The Van Hieles proposed and developed this learning model as a result of the frustration that they experienced with their learners' poor conceptualization of geometric reasoning. The model proposes five levels of thinking that learners sequence through in order for them to master geometric concepts.

The Van Hiele theory identifies a sequence of five hierarchical levels of geometric thinking. These thinking levels are recognition, analysis, ordering, deduction and rigor. According to the Van Hiele theory, "students move sequentially from one level of thinking to the next [level] as their capability increased" (Gutierrez, Jaime & Fortuny, 1991, p. 237).

The levels of thinking comprise a hierarchical nature. They are logically structured to suggest that learners move from lower to higher levels of thinking in geometry. The current level is a prerequisite for the next level. For example, "the recognition of a figure at Level 1 is an essential prerequisite for Level 2. The consideration of properties at Level 2 will eventually lead to Level 3 understanding where students see relationships between them, i.e., how one or two properties lead to a third" (Pegg, 1992, p. 21). The fourth level leads to conceptual understanding of geometrical proof and development and of theorems and postulates.

Difficulties in teaching geometry persisted in the Van Hieles' years of teaching, despite their change of geometric instruction over the years. Van Hiele (1986, p. 39) chronicled; "in the years that followed, I changed my explanation many times, but the difficulties remained. It always seemed as though I were speaking a different language". They then developed a framework of teaching phases that helped teachers to move their learners from one level to the next. Van Hiele-Geldof (1958, as cited in Fuys, et. al., 1984) stresses that learners cannot progress through the levels of thinking without proper instruction. Hence, it is important that the teachers' instruction is pegged at the appropriate Van Hiele level to enable learners to attain the highest possible level in their learning environments.

The Van Hiele's phases of instruction

Van Hiele (1986) recommends a set of instructional phases that teachers should follow in order to facilitate the students' movement between the Van Hiele levels of geometric thinking. The phases of instruction are: information, guided orientation, explicitation, free orientation and integration. Teachers are advised to guide their learners' geometric conceptualisation by employing these five phases of instruction in their practices (Fuys, et. al., 1984, Mistretta, 2000; Clements & Battista, 2004; Groth, 2005; Ding & Jones, 2007, Serow, 2008; Abdullah & Zakaria, 2011). A description of each phase of instruction is summarised below.

Phase 1: Information/Inquiry

During the information phase, the teacher provides inquiry-based activities in which learners carry out 'experiments' and make inductive reasoning and conjectures with regard to the objects learnt. Furthermore, the teacher introduces the vocabulary and concepts necessary for completing a task and for the learners to become familiar with the working domain through discussion and exploration. Discussion takes place between the teacher and the learners to foreground the objects to be used. It is through this discussion that the teacher discovers how learners interpret the language and then provides the necessary information to bring them to purposeful action and perception. Hence, Crowley (1987) reasons that the teachers engage with activities at Phase 1 so that they "learn what prior knowledge the students have about the topic while [italics added] the students learn what direction further study will take" (p. 5).

Phase 2: Guided Orientation

In the second phase, the teacher guides learners to uncover connections and to identify the focus of the subject matter. At this stage, learners are given the liberty to exchange ideas during classroom discussion. They engage with the concepts in order to begin to develop an understanding of them and the connections between them. This enables the learners to learn by exploring the subject matter during the teacher-guided discussions.

At this stage, the teacher is more involved in the learning processes. She/he directs the learners where she/he wants them to go. For example, the teacher sketches a circle and only labels the closely related concepts, say a diameter, radius, chord and secant. She/he then guides the learners to differentiate between these concepts. More unknown concepts are introduced while the teacher guides the learners to use the proper circle geometry terminology. However, the learners' own language is still acceptable at this stage though the teacher's role is to correct their language into a more accepted technical language. Hence, "much of the material will be short tasks designed to elicit specific responses" (Crowley, 1987, p. 5)

Phase 3: Explicitation

In the course of the third phase, explicitation takes place. Learners learn to verbalise their understandings of the concepts and connections. They become more conscious of the new ideas and express these in accepted mathematical language. The concepts now need to be made explicit, using accepted geometric technical language. The teacher takes care that learners advance the use of technical language with understanding through the exchange of ideas. Hence, learners' new knowledge is formed through experience and integrating this with past knowledge.

Phase 4: Free Orientation

In the free orientation phase, learners are challenged to solve problems. They can now complete complex tasks (related to the concepts at hand) that require a number of steps and can be solved in many different ways. Learners are required to find their own way and locate themselves in the network of relations to complete such tasks. They are now familiar with the learning domain and are ready to explore it. Through problem solving, learners' language develops further as they start to identify hints to assist them. The teacher's role in this phase, according to Clements and Battista (2004, p. 431), is "to select appropriate materials and geometric problems" that require certain levels of thinking to solve geometric problems successfully.

Phase 5: Integration

In the integration phase, learners build an overview of the content studied. The teacher helps them to reflect upon the observations they have made and they begin to understand the overall structure of the concepts and where those structures fit into the scheme of formal mathematics. In the same vein, learners summarise the new understanding of the concepts involved and incorporate the appropriate language of the new level. It is important that these summaries only include what the learners already know – no new material should be introduced at this phase. The purpose of instruction is now clear to the learners; hence, there is less and less assistance from the teacher. Learners could, for example, be asked to summarise the content learned over a short period of time, say, a particular theme (e.g. circle geometry).

By the completion of this phase (integration), learners have reached a new level of thinking. This new level of thinking replaces the previous level of thinking and learners are once again ready to repeat the five phases of instruction at the next level of the Van Hiele model of thinking. The cycle keeps on repeating until learners attain the highest possible level of geometric thinking for the content under study.

A UK study of geometry teaching (Royal Society, as cited in Ding & Jones, 2006) concludes that "the most significant contributions in geometry teaching will be generated by the development of good models of pedagogy, supported by carefully designed activities and resources" (p. 45). Hence, instruction planned to nurture development from one level to the next level of thinking should include a sequence of activities, "beginning with an exploratory phase, gradually building concepts and related language, and culminating in summary activities that help students integrate what they have learned into what they already know" (Van Hiele, 1999, p. 311).

Methodology

Research goal and question

The main goal of my research was to design, implement and reflect on a 'circle geometry teaching programme' based on the van Hiele phases of instruction.

The fundamental research question is: What are the experiences of selected mathematics teachers when using the Van Hiele phases of instruction in designing and implementing a Grade 11 circle geometry teaching programme?

The sample

The sample for this case study research is located in a Secondary urban school in Oshikoto Region in Namibia. This is a multicultural school in terms of student profile. The students predominantly come from a middle-class economic background. I selected the school primarily for convenience reasons as I had easy and regular access to it.

The sample consists of 3 Grade 11 mathematics teachers and the 18 learners selected from the Grade 11 class doing Mathematics Higher Level. There are currently five Grade 11 and five Grade 12 classes at the school. One Grade 11 class and one Grade 12 class take Mathematics Higher Level and the rest take Mathematics Ordinary Level. These two levels differ in that the Higher Level covers a major part of circle geometry while the Ordinary Level covers only the basics of circle geometry. Consequently, I selected the Higher Level Grade 11 learners and their three mathematics teachers for the study because their curriculum included sufficient circle geometry.

Orientation of the study

This qualitative case study case study is oriented within an interpretive paradigm. According to Cohen, Manion and Morrison (2007, p. 21), the central endeavour in the context of the interpretive paradigm is "to understand the subjective world of human experience. To retain the integrity of the phenomenon being investigated, efforts are made to get inside the person's head and understand from within". Jackson (1995) suggests that the nature of the interpretive paradigm "stresses the importance of interpretation individuals put on their actions and reactions to others" (p. 9). It is by this virtue that the interpretive paradigm is most suitable for this study.

This research project aims to investigate the teachers' 'experiences'. The best possible way to do that is to provide rich descriptions of the phenomenon and, if possible, to develop some explanations for it (Ellis, 1992). Therefore, employing the interpretive paradigm, I located myself accordingly to engage deeply with the data and analyse the participants' experiences with designing and implementing a circle geometry teaching programme using the Van Hiele phases of instruction as a conceptual framework. I interacted with the Grade 11-12 mathematics teachers over a period of three months. I observed the actual implementation of the teaching programme and engaged the participating teachers in lesson reflections on the whole implementation process.

Method

When I explained the Van Hiele theory to the teachers during the introductory workshop, I started with the levels of thinking as the phases of instruction are embedded within these levels. I explained the features of each level up to the third level of thinking (the abstract level). "At level 3, students can follow a short proof based on properties learned from concrete experiences, but they may not be able to derive such proofs themselves" (Senk, 1989, p. 310).

As the teaching programme had to be designed for secondary school learners, it was only necessary to design it with criteria up to the third level of thinking. In fact, "the goal of most high school geometry course is to have students reasoning at the deductive (abstract) [italics added] level by the end of the year" (Groth, 2005, p. 28). The three teachers and I then designed the following circle geometry teaching programme in alignment with the Van Hiele phases of instruction.

Table 1: The Circle Geometry Teaching programme

Guidelines for the teacher

[2]The intervention is sequenced using the Van Hiele phases of instruction: information, directed orientation, explicitation, free orientation, and integration.

Activity and Phase	Teacher's Instruction/Activity Description
A. **Circle geometry concepts**	1.1.1 Pupils work through simple constructions in their note books using a mathematical set. Instructions include:
Information and Directed Orientation Phases **Activities 1**: Angles in a semi-circle, Tangent of the circle, and tangents from an external point.	a) Construct a circle with centre 0. b) Draw a diameter and name it AB at the point of contact with circumference of the circle. c) Draw two chords, from A and B and let them intersect at point C on the circumference.
Explicitation and Free Orientation Phases →	d) Measure $\angle$ACB. Draw another pair of chords from A and B and let them intersect at C. Measure $\angle$ACB. What did you notice? What conclusions can you draw from these angles?
Integration Phase	At this stage, the pupils should notice that an angle in a semi-circle equals 90 º. The teacher should ensure that the pupils master the correct technical language at this stage such as: semi-circle (the teacher should not sketch any figure at this stage; rather inspect pupils' constructions and ensure they all follow instructions accurately).
Directed Orientation Phase **Activities 2:** Angles between a radius and a tangent of the circle	2.1.1 Construct another circle with centre O. a) Draw a radius and name it OT. Draw a tangent to the circle at T and name it ATB.
Explicitation and Free Orientation Phases	b) Measure $\angle$OTB and $\angle$OTA. What did you notice?
Activities 3: Tangent of the circle, and tangents from an external point **Integration**	c) Draw another tangent from point A on the opposite side of the circle. Draw a radius to this tangent and mark the point of this tangent and mark the point of intersection P hence, the tangent APQ. d) Measure $\angle$OPA and $\angle$OPQ. What did you notice? e) Draw chord PT and measure the lengths of sides AP and AT. What did you notice? What conclusions can you draw from these angles?

Directed Orientation and Explicitation Phases

At this stage, pupils have mastered the concepts that angles formed between the radius and the circle equal 90º. Hence, the tangent and the radius of the circle are perpendicular. The teacher helps them to conclude that tangents from the same point are equal in length.

Activities 4: Angles subtended by the same arc are equal; angles in the same segment are equal

3.1.1 The teacher is aware that the pupils' level of thinking has improved if not increased; hence, instruction should also be at a high level. At this stage, only the correct geometric language should be accepted.

Free Orientation

The teacher should assist the pupils where necessary, for example, by sketching figures Pupils are instructed to:

a) In figure 1, measure ∠ACB, ∠ADB and ∠AEB.
Now measure angle AOB. What did you find?

Teacher: In figure 1, ∠AOB is an angle at the centre of the circle subtended by arc AB.
Angels ACB, ADB and AEB are at the circumference of the circle subtended by arc AB (show arc AB to the pupils).

Integration Phase

b) In figure 2, reflex angle AOB is at the centre of the circle subtended by the major arc AB (show the major arc AB). Angle ACB is an angle at the circumference of the circle, subtended by the same major arc AB. Measure reflex angle AOB and ∠ACB. What did you notice?

c) Compare with the first circle. (You said earlier that the angle in a semi-circle equals 90º. (Is this angle also on the circumference? Is so, what is the size of the angle at the centre of the circle?)

Information and Directed Orientation

Let pupils refer to Figure one and introduce the concept of angles subtended by the same arc. Draw chord AB and talk about angles in the same segment.
Activity: Find the size of the angles c, d, e and give a reason for your answer.
At this stage, pupils have mastered the analysis level (Van Hiele Level 2). They are now prepared and ready to take on thinking of the third level of thinking. This means that the teachers instruction will also be at Level 3 (higher than before), and so do the activities.

	4.1.1 The teacher directs the lesson toward the concepts of cyclic quadrilateral. She/he presents pupils with ready drawn figures as shown below. [The diagrams below are not to scale. O is the centre of the circle].
Explicitation and Free Orientation Phases	The teacher starts by drawing a circle on the chalkboard with two opposite angles ABC and ∠ADC subtended by the same chord (diameter AB). The idea here is to conceptualise concepts of cyclic quadrilateral by realising that angles in opposite segments are supplementary.
Integration	Pupils are instructed to: a) In each diagram, fill in the sizes of the angles at O and at B and D. b) Calculate the sum of∠B +∠D. What did you find? c) What is the sum of∠BAD and ∠BCD? Give a reason for your answer. 5.1.1 Draw a cyclic quadrilateral ABCD. a) Measure all the angles of the cyclic quadrilateral. b) Find the sum of the two pairs of opposite angles. c) What can you conclude about angles in cyclic quadrilaterals? In the following activities, require the pupils to give reasons for their answers using complete sentences in the correct geometric language. Pupils should be operating at VHL3 in order for them successfully complete this activity.

[2]The teaching sequence is adapted from Serow (2008, pp. 448-449). Activities adapted from Courtney-Clarke and Coulson (2008, pp. 203-213).

Findings

The data analysis supported four central claims:

Claim 1: All three participating teachers used and implemented **all** the five Van Hiele phases of *instruction* in their lessons that I observed.

The three teachers' lesson presentations used the common programme on circle geometry developed jointly with them. They either followed it exactly or aligned it to their own instructions and activities.

All three teachers explicitly went through all five Van Hiele phases of instruction during their lessons and also instructed the learners according to the main features of the van

Hiele phases of instruction. For example, during the *information phase*, the teacher is expected to inform the learners about the content to be taught and learns about their existing knowledge. Teacher A informed the learners that they were going to learn about circle geometry and wrote 'Circle Geometry' on the chalkboard. She then asked them about the circle geometry concepts before she instructs them to construct a circle, draw and label as many circle geometry concepts as they could possibly remember.

During the directed *orientation phase*, (which is to direct the learners' learning activities by providing well-planned tasks that build on the established prior knowledge during the information phase), Teacher B, after establishing learners' prior knowledge circle geometry concepts instructed them to *"construct a circle centre O, **diameter** AB. Draw two **chords** from each A and B and let them intersect at a certain point C on the **circumference** of the circle."* The teacher's aim at this point was to see how the learners applied the concepts (**in bold**) learnt in Phase 1 to construct the required circle.

During the *explicitation phase* (in which the learners and the teacher engaged in a discussion about the content whereby the teacher is expected to accept the learners' own language), Teacher C, for example, engaged the learners in a discussion about the diameter of the circle. The learners defined the diameter as any line that passed through the centre of the circle with its endpoints on the circumference of the circle. To test their understanding and further use of geometric language, the teacher gave the learners an activity about the examples and non-examples of a diameter of the circle. She did this by drawing a circle with various lines passing through the centre of the circle and asked the learners to identify the diameter of that circle (See figure 1 below).

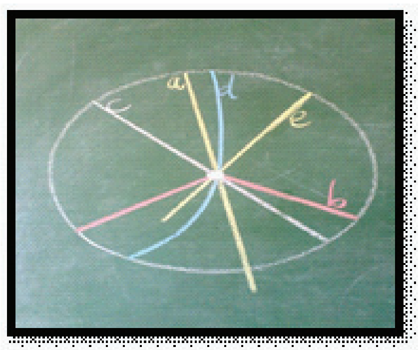

Figure 1: Snapshot of Teacher A's lesson

During the *free orientation phase*, (in which the teacher is expected to select appropriate geometrical problems that learners should complete under his/her observation), Teacher A, for example, provided structured activities for the learners to complete independently while assessing their progress. These activities required the learners to have mastered an advanced knowledge of circle geometry concepts and angle properties to solve them as accurately as possible. Teacher B and C also provided similar activities and observed their learners' progress during the lessons. Here is an example of one of the activities at the free orientation phase.

Activity 1

In figure 4.8 below, points A, B, C and D lie on the circumference of the circle with centre O. Chord AB and CD intersect at X.

$\angle$ BAD = 92º and $\angle$ABC = 57º.

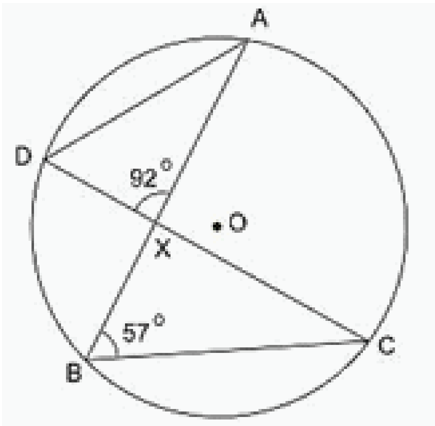

Figure 2: Assessment activity

Instruction: Calculate (giving reasons) the sizes of the following angles:

(a) ADC
(b) BAD
(c) AXC
(d) DCB

During the *integration phase* (which is to encourage the learners to summarise and reflect on the circle geometry knowledge learned) Teacher A for example asked her learners to explain the angle properties of a circle such as: angles in the same segment, angles between tangents and radii of the circle and angle properties of cyclic quadrilaterals. The learners did this orally. Teacher B asked his learners to write a summary of the definitions of the parts of the circle and on the angle properties learnt. Teacher C, on the other hand, drew a circle on the chalkboard and asked the learners one by one to draw the circle geometry concepts that they learned. The teacher also asked the learners to define each concept as they differentiated them from each other.

The above synopsis of the actual implementation of the Van Hiele teaching programme shows how all three teachers, collectively, used the five phases of instruction to teach their lessons. During the interview of her first lesson, I asked Teacher C if she had been aware of using the van Hiele phases during her lesson.

Interviewer: *Did you at all notice if you used the Van Hiele phases during your lesson?*

Teacher C: *Err… Not really that I noticed much coz I don't really… hmm… but I think I have because I started from the beginning. Like you're telling them what the topic is all about and what to do and then you're letting them to apply what you've told them onto the chalkboard to show their working.*

Interviewer: *Are those now the first two phases?*

Teacher C: *Yeah*

Interviewer: *You gave them the information and then you told them what to do?*

Teacher C: *Hmm… and then I let them do it or identify or label – put the labels on the parts of the circle that we learned. I also asked them to give reasons for their answers when I told them to complete the exercises on the chalkboard.*

The teacher acknowledged that she was not sure if she understood the Van Hiele phases/ levels. She could not name them in order from the information to the integration phase but understood them in her own way. The above interview transcript showed that Teacher C used the phases of instruction in her first lesson.

Teacher A also acknowledged that she did not really understand *"this van … theory"* but she used the teaching programme to teach her lessons thoroughly. Teacher B, on the other hand, kept on reminding me that it was I who introduced him to the Van Hiele phases of instruction and he only did at each phase what I instructed him to do. For example, when I asked him about his instructions during the information phase of his first lesson, he said that *"you told me about the phases of instruction. So, I think that the sooner the learners learn about the stationery required in circle geometry the better."* Teacher B further explained that he was just trying to inform the learners [probably at the appropriate phase of instruction]. Teacher B also commented on how he felt about his practice during the explicitation phase. *"I think the explicitation phase is the one that is mostly used by the teachers to thoroughly explain the content to the learners."*

I was interested in whether these participating teachers really understood the phases of instruction as the Van Hieles themselves presented them in their theory, or had they adopted their own parallel framework to that of the Van Hieles. When I asked the teachers if they used the phases of instruction to teach, they said that they did, but did not particularly mention the phases per se i.e. in the information phase, I did this and that. They rather sequenced what they did first, second, third, etc. When I analysed this sequence, I found that it was very similar to the five Van Hiele phases of instruction (Fig.3).

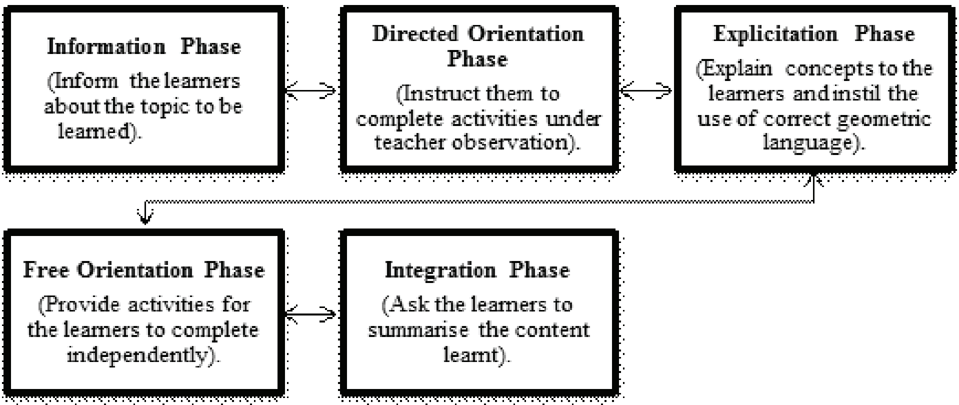

Figure 3: Appropriate visualisation of teachers' understanding of the van Hiele phases of instruction.

Claim 2: The teachers navigated quite freely from one phase of instruction to the next, but also returned to the earlier phases.

During the classroom observations, I noticed that the teachers moved backwards and forwards between the phases of instruction and yet could still stay on track with the programme. For example, when operating in the free orientation phase, the teachers could refer back to earlier phases of information and directed orientation phase, and sometimes even to the explicitation phase to clarify missing/unclear concepts. They would then go back to the free orientation phase and continue with what they were doing.

A typical example is from Teacher B's first lesson when he asked the learners to construct a circle which should be a set of points that were 5cm from a fixed point. When the teacher noticed how puzzled his learners looked, he decided to define the circumference of the circle (explicitation phase) as a set of points which are a fixed distance (pointed at a radius) from a fixed point (centre of the circle). The teacher explained the concept while providing sufficient information to supplement his explanation. He then proceeded with his initial instruction when he noticed that the learners were coping.

The assessment activities that the teachers gave to the learners during the classroom observations showed how the teachers required their learners to navigate between the phases of instruction to find solutions to geometric problems. Teacher A, for example, asked the learners to write down all the circle geometry concepts that they could see in figure 4 (activity 2 below). The first two instructions that the teacher gave to the learners where not necessarily aimed at the purpose of the activity – the teacher saw fit to take the learners back to the information phase (to recall concepts and angle properties of circles) before she allowed them to operate at the explicitation phase again (See Fig. 4). After the teacher was satisfied with the learners' responses, she gave them the remaining instruction which was to "calculate the size of SUT. Show all your working and give a reason for your answer."

Activity 2

In figure 4 below, points S, U and T lie on the circumference of the circle centre; O. RS and RT are tangents to the circle at S and T respectively. SRT equals 40º.

Instructions:

1) *Write down all the circle geometry concepts that you can see i.e. centre O.*
2) *Write down all the circle geometry theorems that we have learned.*
3) *Calculate the size of SUT. Show all your working and give a reason for your answer.*

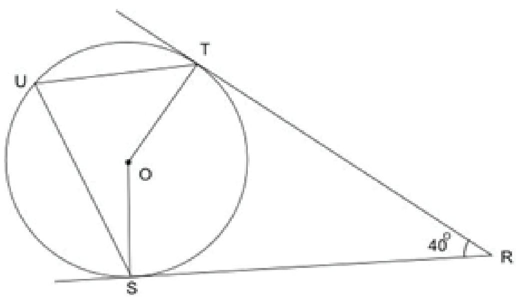

Figure 4: Assessment activity

Activity 2 above required knowledge of angle properties which in turn needed knowledge of circle geometry concepts. Teacher A deemed it necessary to navigate between the phases of instruction to give clear instructions to the learners. Hence, the teacher's role during the explicitation phase is "to bring the objects of study i.e. Circle geometry concepts and theorems to an explicit level of awareness by leading students' discussion of them in their own language" (Clements & Battista, 1992, p. 431). The three teachers encouraged the learners to actively engage in assessment activities both as individuals and as a group. They did not accept the learners' use of informal language as they replaced it with the correct geometric language.

Furthermore, in the introductory workshop, Teacher A advised that I should inform Teacher C on the flexibility of the Van Hiele phases of instruction. *"Don't forget to explain to Teacher C that when you are teaching, you always start with the information phase then the directed orientation phase and so forth. But when you are explaining something, the order of the phases does not matter. You can always go back to the information or any other phase to make the learners understand."* Teacher C in return navigated well between the explicitation and information phases during the task on examples and non-examples of a diameter of the circle (Figure 1).

Claim 3: The phases of instruction are seen by the teachers as a good pedagogical tool or template for planning and presenting lessons.

The circle geometry teaching programme was an effective tool for sequencing activities that involved circle geometry for this study. The programme enabled the teachers to align the activities to the five phases of instruction. The learning activities that teachers used for explanation on the chalkboard, for example, were approached in a manner which is supported by the Van Hiele phases of instruction. For example, Teacher B drew a circle on the chalkboard when he introduced circle geometry to his class. The teacher started by labelling the parts of the circle in alphabetical order. He labelled the centre of the circle as (a), the circumference (b), the radius (c) etc. as he needed these to inform parts that followed. For example, the diameter passes through the *centre* and is twice the *radius*, the chord has its endpoints on the *circumference*, and so forth. This is how the phases of instruction also support each other. The information informs all the other phases of instruction and vice versa.

Towards the end of the introductory workshop, Teacher A expressed that this was a good teaching programme which might work with other sections as well. She suggested that we could also use the teaching programme to plan lessons on Graphs and Functions in which our learners also experience difficulties. *"These phases of instruction can also be used in Graphs and Functions and also Transformations chapters where most of our learners also have difficulties."* Teacher C commented that she could use the phases of instruction across the curriculum. *"I think I can also use these phases in Life Science."*

I noticed during the classroom observations that the teachers were very confident about their own practices. I also noticed when I reviewed the videotapes that as the teachers' lessons progressed; they were not sidetracked by my visits in their classrooms. Various comments during the reflective interviews also showed how effective this teaching programme had been for them. For example, when I asked Teacher A how she felt about the teaching programme during the free orientation phase, she stated, *"I feel that this teaching programme can do some good to every mathematics teacher especially the teachers who are teaching for the first time. It really helped me to plan my lessons and activities even though I kept calling you to guide me."*

Even Teacher C, who was very nervous at the beginning of her first lesson, revealed how effective the teaching programme had been to her pedagogical practices. *"Have you noticed that these learners only need a little push and then their thinking is already in order? ... I cannot believe I was so nervous about something so interesting like this teaching programme."*

Claim 4: The majority of the learners followed instructions and seem to obtain the answers faster than expected.

Commonly, good practice enhances competent outcomes. All the teachers claimed that they did their part, and the learners performed very well during the assessment activities – often faster than they expected.

In addition to the overall learner involvement, the learners remained on-task for the entirety of the learning time and throughout each activity. They conversed at length with each other and the teachers as they progressed from informal to formal language usage and reasoning. During the free orientation phase, "the students engage in more open-ended activities that can be approached by several different types of solutions (Teppo, 1991, p. 212). Here is one example of the learners' activity that required them to use the correct geometric language and reasoning:

In circle O, diameter $\overline{AB}$, radius $\overline{OC}$, and chord $\overline{BC}$ are all drawn. If $\angle$ AOC = 50 , find OCB. Show all your working.

The learners were expected to find their own method of solving the problem as no further information was given (this is a typical free orientation phase activity). The learners were placed in a situation where they were required to use various geometry concepts and angle properties of circles in order to compute a solution to this problem. Learners applied various methods to solve the problem. I observed that learners' reasoning about their solutions to problems was very profound.

I analysed the data collected for this study by writing vignettes of actual lesson presentations of the three participating teachers. The findings of this reaserch were expressed in terms of asserted claims. These claims were then used to reveal the three teachers' experiences with the designing and implementing a circle geometry teaching programme that used the Van Hiele phases of instruction as a conceptual framework.

Limitations of the study

This study used a very small sample and its findings cannot be generalised due to the following reasons:

- The research was conducted in a single urban secondary school with a multicultural student and teacher profile, predominantly from a middle-class economic background. The results can, therefore, not be generalised as there could be different findings from a school in a rural area, and/or a school in any other town in Namibia.

- Time constraints and over-commitment of the teachers made it very challenging for me to secure consistent participation. Also we could not finish teaching all of the intended content of the teaching programme, due to its length and the limited time allocation I had with the teachers.

Conclusion

This was a succinct synopsis of the research study conducted during the 2012 academic year. The purpose this research was the planning, designing and implementing of a circle geometry teaching programme using the Van Hiele phases of instruction as a conceptual framework. This programme was aimed at improving classroom instruction and learner participation during circle geometry lessons. The findings indicate that the designed teaching programme can be used to positively enhance effective teaching and learning and, most importantly, to improve teachers' instruction.

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